湮灭算符：$\hat{a}=(\frac{m\omega}{2\hbar})^{\frac{1}{2}}(\hat{x}+\frac{i}{m\omega}\hat{p})$
产生算符：$\hat{a}^{\dagger}=(\frac{m\omega}{2\hbar})^{\frac{1}{2}}(\hat{x}-\frac{i}{m\omega}\hat{p})$

证明对易关系：$[\hat{a},\hat{a}^{\dagger}]=1$

$$\begin{aligned}[\hat{a},\hat{a}^{\dagger}]&=[(\frac{m\omega}{2\hbar})^{\frac{1}{2}}(\hat{x}+\frac{i}{m\omega}\hat{p}),(\frac{m\omega}{2\hbar})^{\frac{1}{2}}(\hat{x}-\frac{i}{m\omega}\hat{p})]\\&=\frac{m\hbar}{2\hbar}[\hat{x}+\frac{i}{m\omega}\hat{p},\hat{x}-\frac{i}{m\omega}\hat{p}]\\&=\frac{m\hbar}{2\hbar}\{[\hat{x},\hat{x}]-[\hat{x},\frac{i}{m\omega}\hat{p}]+[\frac{i}{m\omega}\hat{p},\hat{x}]-[\frac{i}{m\omega}\hat{p},\frac{i}{m\omega}\hat{p}]\}\\&=\frac{m\hbar}{2\hbar}\{-\frac{i}{m\omega}[\hat{x},\hat{p}]+\frac{i}{m\omega}[\hat{p},\hat{x}]\}\\&=\frac{m\hbar}{2\hbar}\cdot\frac{2\hbar}{m\omega}\\&=1\end{aligned}$$

对于一维线性谐振子：

$$\xi=\sqrt{\frac{m\omega}{\hbar}}x=\alpha x$$

从而得出：

$$\hat{a}=\frac{1}{\sqrt{2}}(\xi+\frac{\partial}{\partial\xi})$$

$$\hat{a}^{\dagger}=\frac{1}{\sqrt{2}}(\xi-\frac{\partial}{\partial\xi})$$

将$\hat{a}$作用于谐振子哈密顿算符的第n个本征态ψ_n，可以得到：

$$\hat{a}\psi_n=\frac{1}{\sqrt{2}}(\xi+\frac{\partial}{\partial\xi})\psi_n$$

再根据：

$$\xi\psi_n=\sqrt{\frac{n}{2}}\psi_{n-1}+\sqrt{\frac{n+1}{2}}\psi_{n+1}$$

$$\frac{d}{d\xi}\psi_n=\sqrt{\frac{n}{2}}\psi_{n-1}-\sqrt{\frac{n+1}{2}}\psi_{n+1}$$

最终得到：

$$
\begin{aligned}
\hat{a}\psi_n &= \frac{1}{\sqrt{2}}\xi\psi_n + \frac{1}{\sqrt{2}}\frac{\partial}{\partial\xi}\psi_n \\
&= \frac{1}{\sqrt{2}}(\sqrt{\frac{n}{2}}\psi_{n-1} + \sqrt{\frac{n+1}{2}}\psi_{n+1} + \sqrt{\frac{n}{2}}\psi_{n-1} - \sqrt{\frac{n+1}{2}}\psi_{n+1}) \\
&= \frac{1}{\sqrt{2}} \cdot 2 \cdot \sqrt{\frac{n}{2}}\psi_{n-1} \\
&= \sqrt{n}\psi_{n-1}
\end{aligned}
$$

同理:

$$\hat{a}^{\dagger}\psi_n = \sqrt{n+1}\psi_{n+1}$$